



**ST STITHIANS BOYS' COLLEGE**  
**ADVANCED PROGRAMME MATHEMATICS**  
**GRADE 12**  
**JULY 2009**



Time: 3 hours

300 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 7 pages. Please check that your question paper is complete.
2. This question paper consists of two sections.  
**SECTION A: CALCULUS AND ALGEBRA (200 marks)**  
**SECTION B: FINANCE AND MODELLING (100 marks)**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All answers are to be given correct to 2 decimal digits where necessary.
5. All necessary calculations must be clearly shown and writing should be legible.
6. Diagrams have not been drawn to scale.
7. Write all your answers in the separate Answer Books provided. Each section is to be answered in a separate booklet as indicated on the booklet cover.
8. The Formula sheet is attached at the back of your question paper.

**SECTION A: CALCULUS AND ALGEBRA (200 marks)****QUESTION 1: Solve for  $x$ ,  $x \in \mathbf{R}$  : (28 marks)**

$$1.1 \quad |3x^2 - 2x| = 1 \quad (7)$$

$$1.2 \quad 3e^x - \frac{2}{e^x} = 1 \quad (6)$$

$$1.3 \quad \ln(e^{2x} - 12) = x, \text{ correct to 2 decimal digits} \quad (7)$$

$$1.4 \quad \frac{8}{x+6} \leq 2 \quad (8)$$

**QUESTION 2: No calculators to be used in this question (18 marks)**

$$2.1 \quad \text{Rewrite } \frac{(1-4i)^2}{i+1} \text{ in the form } a + bi \quad (3)$$

$$2.2 \quad \text{Solve for } x \in \mathbf{C} \text{ given that } x = 1 - 3i \text{ is a solution to } x^3 + x^2 + 4x + 30 = 0 \quad (9)$$

$$2.3 \quad \text{For how many values of } n \text{ is } i^{3n+2} \text{ a real number if } 0 \leq n \leq 155? \\ \text{Show all your working.} \quad (6)$$

**QUESTION 3: (10 marks)**

The function  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} \frac{a}{x} & \text{if } x \geq 1 \\ b - 2x & \text{if } x < 1 \end{cases}$$

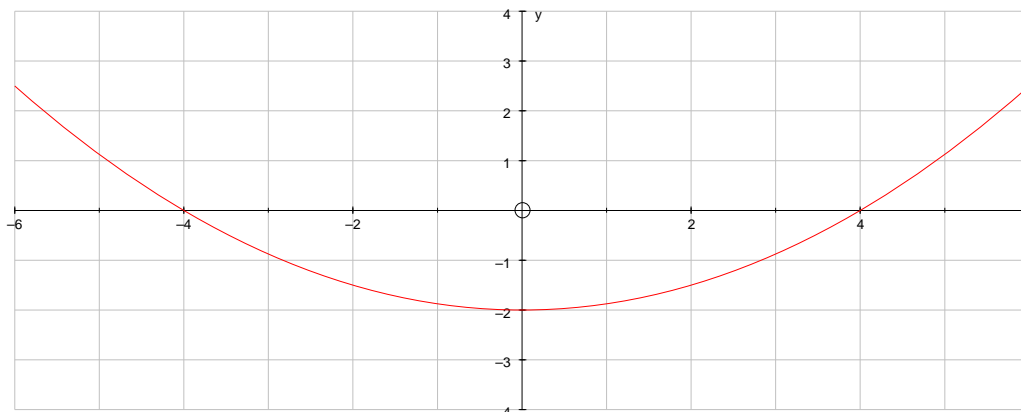
Determine the value(s) of  $a$  and  $b$  if  $f(x)$  is differentiable at  $x = 1$  (10)

**QUESTION 4: (12 marks)**

Sketch the graphs  $y = x^3 - x^2 - 4x + 4 = (x-2)(x+2)(x-1)$  and  $y = x$  to show that the equation  $x^3 - x^2 - 4x + 4 = x$  has 3 solutions and use Newton's method to find the largest solution correct to 3 decimal places. (12)

**QUESTION 5: (8 marks)**

Refer to the diagram showing the graph of  $y = f(x)$ .



Draw sketch graphs of the following and state the domain and range for each graph:

5.1  $y = |f(x)|$  (4)

5.2  $y = 2 \times f(x+2)$  (4)

**QUESTION 6: (68 marks)**

6.1 Without using a calculator, determine:

6.1.1  $\lim_{x \rightarrow \infty} \frac{5 - 3x - 2x^2}{(2x - 3)^2}$  (5)

6.1.2  $\lim_{x \rightarrow 0} \frac{x^2}{\tan 2x \cdot \tan 3x}$  (7)

6.1.3  $\lim_{x \rightarrow 1} \frac{3^{2x} - 9}{3^x - 3}$  (4)

6.2 Differentiate  $f(x) = \frac{-3}{x^2}$  from first principles (10)

- 6.3 Determine  $\frac{dy}{dx}$  :
- 6.3.1  $y = \frac{5}{x}(1 - \sqrt{x})$  (6)
- 6.3.2  $x^3 + y^3 = xy$  (9)
- 6.3.3  $y = \sqrt{x^2 + 1} + 4(3x + 1)^5$  (6)
- 6.3.4  $y = \sqrt{x} \cos^3 2x$  (6)
- 6.4 Given that  $f(x) = \frac{1}{2x^2}$  determine a formula for  $f^n(x)$  (7)
- 6.5 Determine the gradient of the graph of  $y = 2 \tan^2\left(\frac{\pi}{2} - 3\theta\right)$  when  $\theta = \frac{\pi}{4}$ . (8)

**QUESTION 7: (30 marks)**

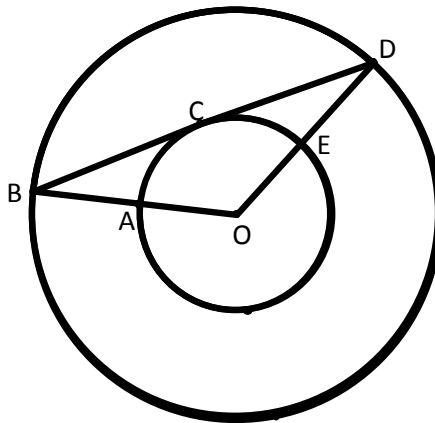
Given that  $h(x) = \frac{2x^3}{x^2 - 4}$

- 7.1 Give the domain of  $h(x)$  (2)
- 7.2 Determine the intercepts that the graph makes with the axes (2)
- 7.3 Determine the co-ordinates of all stationary points (11)
- 7.4 Determine the equations of all asymptotes (6)
- 7.5 Draw a neat sketch of  $h(x)$ , showing the co-ordinates of the intercepts with axes, asymptotes and turning points. (9)

**QUESTION 8: (15 marks)**

Two concentric circles are shown with centre O. BD is a tangent to the smaller circle at C. OAB and OED are straight lines. BCD is a tangent at C with  $BC = CD$ .  $\hat{AOE} = \frac{5}{9}\pi$  radians.

$OE = 3$  cm and  $AB = 2$  cm.



8.1 Determine the area of the grey segment of the larger circle cut off by  $BD$ . (7)

8.2 Determine the black area  $ACB$  (8)

**QUESTION 9: (11 marks)**

Use Mathematical Induction to prove that:

$$\log x + 2\log x + 3\log x + \dots + n\log x = \frac{n}{2}\log x^{n+1} \text{ for all } n \in N \quad (11)$$

**SECTION B: FINANCE AND MODELING (100 marks)****QUESTION 1: (9 marks)**

The sequence  $8; 6; b; -66; -142$  is generated by the formula  $T_n = a.T_{n-1} - 4T_{n-2}$ .

Determine the possible values of  $a$  and  $b$  (9)

**QUESTION 2: (10 marks)**

An investment now is worth four times as much in 8 years time. Calculate:

2.1 The effective annual interest rate. (5)

2.2 The nominal annual rate if compounding occurs quarterly. (5)

**QUESTION 3: (15 marks)**

Mr Jones deposits R1 500 now, R2 000 in 3 year's time and R2 500 after a further 6 years. The interest rate he is offered is 9,5% p.a., compounded monthly. Mr Smith deposits R3 000 now in a different account offering a constant annual interest rate.

3.1 How much will Mr Jones have after 12 years? (8)

3.2 What effective annual interest rate will Mr Smith require such that he will receive the same amount saved as Mr Jones after 12 years? (7)

**QUESTION 4: (24 marks)**

Mr Van Der Merwe purchases a new bus for his son's rugby club for R950 000. They plan to use it for 5 years and then trade it in for a new model.

Inflation is estimated at 9% p.a. whilst the value of the bus after 5 years is expected to be R473 499,81.

The club sets up a sinking fund in order to cater for the purchase of a new bus in 5 years time as well as pay for a major service costing R9 500 after 3 years.

The interest payable is calculated at 11% p.a., compounded monthly.

- 4.1 What will the cost of a new bus be in 5 years time? (5)
- 4.2 What is the annual rate of depreciation of the old bus on a reducing balance basis? (5)
- 4.3 Determine the monthly payment into the sinking fund if payments start immediately and finish at the end of 5 years. (14)

**QUESTION 5: (30 marks)**

A loan of R2 400 000 is to be repaid by means of monthly payments, starting in one months time. The interest on the outstanding balance is 13,5% p.a., compounded monthly.

- 5.1 If the loan is fully repaid after 15 years, calculate the monthly payment. (8)
- 5.2 If the loan is to be repaid by monthly payments of R32 000, write down a difference equation for the outstanding balance after  $n$  payments. (6)
- 5.3 5.3.1 How long will it take to amortise the loan if the monthly payments are R28 000 a month? (8)
- 5.3.2 What is the amount of the final payment? (8)

**QUESTION 6: (12 marks)**

Mr Brown takes out a loan of R1 500 000. He agrees to repay the loan by means of monthly payments of R35 684,90 starting one month after the granting of the loan. Interest on the loan is charged at 15% p.a compounded monthly. The loan is amortised at the end of 5 years.

Mr Brown runs into financial difficulties at the end of two years and thus misses the payments due at the end of the 25<sup>th</sup>, 26<sup>th</sup>, 27<sup>th</sup> and 28<sup>th</sup> months. To compensate for this, his monthly payments increase from the end of the 29<sup>th</sup> month. Determine his new monthly payment.

(12)