①

Prelim Advanced Prog Maths

1. $\sum_{i=1}^{n}(4i+7) = 2n^2+9n$

$11+15+19+\ldots = 2n^2+9n$ ✓

Prove true for $n=1$

LHS 11 RHS $2(1)^2+9(1)=11$ $\therefore$ True for $n=1$ ✓✓✓

Assume true for $n=k$

$11+15+19+\ldots+(4k+7) = 2k^2+9k$ ✓✓

Prove true for $n=k+1$

$\underbrace{11+15+19+\ldots+(4k+7)}+(4(k+1)+7) = 2(k+1)^2+9(k+1)$ ✓ ✓

LHS $2k^2+9k+4k+4+7$

$2k^2+13k+11$ ✓

RHS $2k^2+4k+2+9k+9$

$2k^2+13k+11$ ✓

Story ✓   ⑫

2.1 $(\ln x)^2 = \ln e^2 + \ln x$

$k^2 = 2+k$ ✓

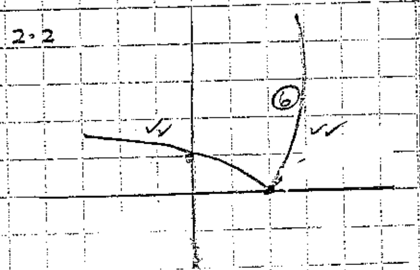
$(k-2)(k+1)=0$ ✓

$\ln x = 2$ ✓ or $\ln x = -1$ ✓

$e^2 = x$ ✓ $e^{-1} = x$ 0.367

7,389 $\frac{1}{e} = x$ ✓   ⑥

2.2



$y = |2-e^x|$

let $x=0$: $\underline{y=1}$ ✓

let $y=0$: $2=e^x$, $\underline{0.69 = x}$ ✓

3.1 $(x-(3-5i))(x-(3+5i))$ ✓✓

$(x-3+5i)(x-3-5i)$ ✓

$= (x-3)^2+25$ ✓

$= \underline{x^2-6x+34}$ ✓✓   ⑥

3.2 $(x^2-6x+34)(x+5) = 0$ ✓

$x = 3-5i$ ✓

$x = 3+5i$ ✓

$x = -5$ ✓   ④

②

4. $\dfrac{1003}{1002\times 1001^2} = \dfrac{A}{1002}+\dfrac{B}{1001}+\dfrac{C}{1001^2}$

$\dfrac{x+2}{(x+1)(x)^2}$ ✓ $= \dfrac{A}{x+1}+\dfrac{B}{x}+\dfrac{C}{x^2}$ ✓✓

$x+2 = Ax^2 + Bx(x+1) + C(x+1)$ ✓✓

let $x=0$: $2=C$ ✓✓

let $x=-1$: $1 = A(-1)^2$, $\underline{1=A}$ ✓✓

let $x=1$: $3 = 1(1)^2 + B(1)(2) + 2(2)$

$3 = 2B+5$

$-2 = 2B$

$\underline{-1=B}$ ✓✓   ⑪

5.1 $x < 9$ ✓

Range $-1 < y \leq 4$ ✓✓✓   ④

5.2 $f(1) = 4$ ✓

$\lim_{x\to1^+} f(x) = 5-1 = 4$ ✓ $\lim_{x\to1^-} f(x) = 3+1 = 4$ ✓ $\therefore \lim_{x\to1} f(x) = 4$ ✓

now $f(1) = \lim_{x\to1} f(x) = 4$ ✓✓ $\therefore$ continuous   ⑥

5.3 jump ✓✓   ②

5.4 $\lim_{x\to7^-} f'(x) = \lim_{x\to7^-}(0) = 0$ ✓✓   Continuous ✓

$\lim_{x\to7^+} f'(x) = \lim_{x\to7^-} 2(x-7) = 0$ ✓

$\therefore \lim_{x\to7^-} f'(x) = \lim_{x\to7^+} f'(x)$ ✓✓   ⑥

$\therefore$ Differentiable

6.1 $\lim_{x\to0} \dfrac{\sin 3x}{x\cos 2x}\times\dfrac{3}{3}$ ✓✓ $= \lim_{x\to0}\dfrac{3\sin 3x}{3x\cdot\cos 2x}$ ✓✓ $= 3.1.1 = 3$ ✓✓   ⑥

6.2 $\lim_{x\to\infty}\dfrac{x^2+1}{2-3x-4x^2} = \lim_{x\to\infty}\dfrac{1+\frac{1}{x^2}}{\frac{2}{x^2}-\frac{3}{x}-4}$ ✓✓ $= -\frac{1}{4}$ ✓✓   ④

(3)

6.3 $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

$= \lim_{x\to 4} \frac{(\sqrt{x+5}-3)}{x-4}\frac{(\sqrt{x+5}+3)}{\sqrt{x+5}+3}$

$= \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)} = \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{6}$ (6)

7.1 $f(x) = x^3(3x+1)$

$f'(x) = 3x^2(3x+1) + 3(x^3)$ (4)

7.2 $g'(x) = \frac{3x^2(x^2+2) - 2x(x^3-1)}{(x^2+2)^2}$ (6)

$f(x) = \sin(\tan(2x))$

$g(x) = \sin x . \tan 2x$

1) $f'(x) = \cos(\tan(2x)) \cdot \sec^2(2x) . 2$

$f'(\pi) = \cos(\tan 2\pi) \sec^2(2\pi) . 2$

$= \cos 0 . \frac{1}{\cos^2 2\pi} . 2$

$= 1 . \frac{1}{1^2} . 2$

$= 2$ (5)

II) $g'(x) = \cos x . \tan 2x + \sec^2 2x . 2 . \sin x$

$g'(\pi/3) = \cos\frac{\pi}{3} . \tan\frac{2\pi}{3} + \frac{1}{\cos^2\frac{2\pi}{3}} . 2 . \sin\frac{\pi}{3}$

$= \frac{1}{2} . (-\sqrt{3}) + 4 . 2 . \frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{3}}{2} + 4\sqrt{3}$

$= \frac{7\sqrt{3}}{2}$ (7)

9. $f(x) = (2-3x)^{20}$

$f^{n}(x) = \frac{20!}{(20-n)!}(2-3x)^{20-n} \times (-3)^n$

9.1 $f'(x) = 20(2-3x)^{19} \times -3$

$f''(x) = 20 \times 19 (2-3x)^{18} \times (-3)^2$

$f'''(x) = 20 \times 19 \times 18 (2-3x)^{17} \times (-3)^3$ (6) (6)

(4)

10.1 $x^m y^n = k$

$mx^{m-1} . y^n + \frac{dy}{dx} ny^{n-1} . x^m = 0$ (6)

$\therefore \frac{dy}{dx} = \frac{-mx^{m-1} . y^n}{ny^{n-1} . x^m} = -\frac{my}{nx}$

10.2 Let $m=2$, $n=3$

If $x=1$: $(1)^2(y)^3 = -8$

$y^3 = -8$

$y = -2$

$\therefore$ point $(1; -2)$ (8)

$m = \frac{-2(-2)}{3(1)} = +\frac{4}{3}$ $\therefore$ $y = \frac{4}{3}x + c$ use $(1; -2)$

$y = \frac{4}{3}x - 3\frac{1}{3}$

11.1 $h(a) = \frac{f(a)}{g(a)}$

$\therefore -\frac{1}{3} = \frac{1}{g(a)}$ (3)

$\therefore g(a) = -3$

11.2 $h'(a) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

$= \frac{2(-3) - 6(1)}{(-3)^2}$

$= \frac{-6-6}{9}$

$= -\frac{4}{3}$ (7)

12. Area of square $= DC^2$

$DC^2 = r^2 + r^2 - 2r^2\cos\theta$

$= 2r^2 - 2r^2\cos\theta$

Area shaded $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ (8)

$\therefore 8\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right) = 2r^2 - 2r^2\cos\theta$

$4r^2\theta - 4r^2\sin\theta = 2r^2 - 2r^2\cos\theta$ $r \neq 0$

$\therefore 4\theta - 4\sin\theta = 2 - 2\cos\theta$

$2\theta - 2\sin\theta + \cos\theta - 1 = 0$

(5)

b) $2\theta - 2\sin\theta + \cos\theta - 1$

(i) $\theta = 1$ $\quad -0{,}1426$ ✓✓ (4)

(ii) $\theta = 2$ $\quad 0{,}765$ ✓✓

c) use $\theta = 1{,}5$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$x_1 = 1{,}5 ✓ - \frac{2(1{,}5) - 2\sin(1{,}5) + \cos(1{,}5) - 1}{2 - 2\cos(1{,}5) - \sin(1{,}5)} ✓✓ ✓✓$$

$= 1{,}412027252$

$x_2 = 1{,}401526969$

$x_3 = 1{,}401379485$

$x_4 = 1{,}40137945 6$

$\underline{\theta = 1{,}4013}$ ✓✓ (8)

13 $\quad f(x) = \dfrac{x^2+4x+3}{x+2}$

13.1 $\quad x+2 - \dfrac{1}{x+2}$ or

$$\begin{array}{r|l} & x+2 \\ \hline x+2 & x^2+4x+3 \\ & \underline{x^2+2x} \\ & 2x+3 \\ & \underline{2x+4} \\ & -1 \end{array}$$ ✓✓ ✓✓ (4)

$= \dfrac{(x+2)^2 - 1}{x+2}$ ✓✓

$= \dfrac{x^2+4x+3}{x+2}$ ✓✓

13.2.1 $\quad f'(x) = 1 + ✓✓ \dfrac{1}{(x+2)^2}$ ✓✓ (4)

13.2.2 $\quad (x+1)^2 > 0$ for all x

$f'(x) \underline{>0}$ for all x

never decreasing ✓✓ (2)

13.2.3 $\quad f''(x) = \dfrac{-1}{(x+2)^3}$ ✓✓ (2)

13.3 vertical asymptote : $x = -2$ ✓✓

horizontal asymptote : ×

oblique asymptote : $y = x+2$ ✓✓

x cut let $y=0$

$x = -3$

$x = -1$ ✓✓

y cut: let $x = 0$

$y = 3/2$ ✓✓

Stationary : $1 + \dfrac{1}{(x+2)^2} = 0$ ✓✓

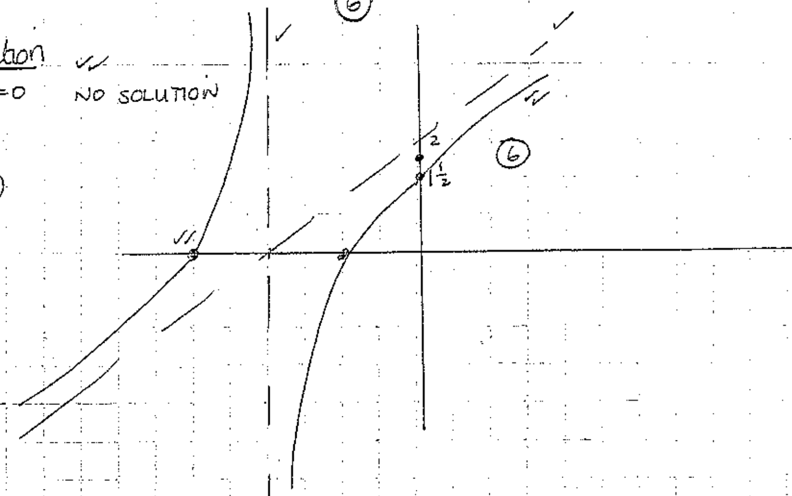
NO SOLUTION

Points inflection ✓✓

$\dfrac{-1}{(x+2)^3} = 0$ NO SOLUTION

(13)

(6)



(6)

14. $AB = \sqrt{(a-3)^2 + (a^2-0)^2}$ ✓✓

$AB^2 = a^2 - 6a + 9 + a^4$ ✓✓

$\dfrac{dL}{da} = 2a - 6 + 4a^3$ ✓✓ (7)

$2a^2 + a - 3 = 0$ ✓ for max | min

14.2 $\quad 2a^3 + a - 3 = 0$ (5)

let $a = 1$ ✓✓

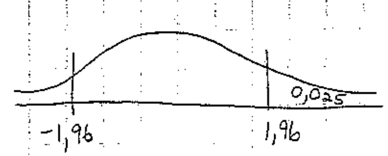
$(a-1)(2a^2 + 2a + 3) = 0$ ✓

$\underline{a = 1}$ ✓ NO SOLUTION

Question 1

1. $H_1: \mu_x \neq \mu_y$ ✓✓ (2)

1.2



$-1{,}96 < z < 1{,}96$ ✓✓✓ (4)

1.3 $\quad z = \dfrac{75{,}5 - 78{,}2}{\sqrt{\frac{100}{30} + \frac{110{,}25}{28}}}$ ✓✓ ✓✓

$z = -1{,}00$ ✓✓

✓✓

Test stat lies in accep region, we accept null hypothesis that there is no signif diff between the means of the two populations

(7)

2.1. $y = A + Bx$

$A = 11,7040$ (4)

$B = 0,1861$

2.2 $B = \dfrac{7(495) - 38(89)}{7(270) - (38)^2} = 0,1861$ (6)

2.3 $r = 0,3782$ (4)

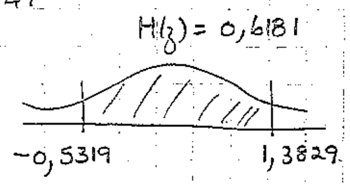
3. $\binom{18}{4}(0,15)^4(0,85)^{14} = 0,1592$ (8)

4. $\dfrac{\binom{4}{0}\binom{16}{5} + \binom{4}{1}\binom{16}{4}}{\binom{20}{5}} = 0,7513$

$\therefore$ 0,2487 of rejecting (12)

5. $\bar{x}$ = 4hours 25 minutes = 265 mins

$\sigma = 47$



$z_1 = \dfrac{240 - 265}{47} = -0,5319$

$z_2 = \dfrac{330 - 265}{47} = 1,3829$

$H(z_1) = 0,2019 \quad H(z_2) = 0,4162$

$\therefore$ $\underline{0,6181}$

$\therefore$ No of cyclists

$= 0,6181 \times 200$

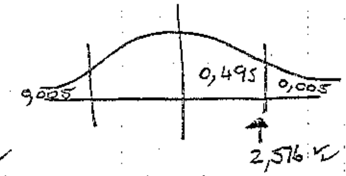
$= 123$ cyclists. (10)

(8)

6. $n = 64$

$\bar{x} = 32,5$

$\sigma^2 = 10,89$



$\mu = 32,5 \pm 2,576\sqrt{\dfrac{10,89}{64}}$

$31,4374 \leq \mu \leq 33,56$ (99% CI) (8)

7. $0,257 = 0,2245 + 1,96\sqrt{\dfrac{0,2245(0,7755)}{n}}$

$0,0325 = 1,96\sqrt{\ldots}$

$0,0165816\ldots = \sqrt{\dfrac{0,2245(0,7755)}{n}}$ (10)

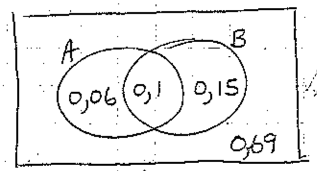
$2,749505414 \times 10^{-4} = \dfrac{0,2245(0,7755)}{n}$

$\underline{633 = n}$

8 $P(A) = 0,16$

$P(B) = 0,25$

$P(A \cap B) = 0,1$



1) $P(A \cup B) = 0,06 + 0,1 + 0,15$

$= 0,31$ (6)

2) $P(A \cap B') = 0,06$ (4)

b) $0,16 \times 0,25 = 0,04$ <u>NOT</u> independent (6)

9. $\frac{1}{k}(2(1)+1) + \frac{1}{k}(2(0)+1) + \frac{1}{k}(2(2)+1) + \frac{1}{k}(2(3)+1) = 1$

$\frac{3}{k} + \frac{1}{k} + \frac{5}{k} + \frac{7}{k} = 1$

$\frac{16}{k} = 1 \quad \therefore \underline{k = 16}$