ST STITHIANS COLLEGE
ADVANCED PROGRAMME MATHEMATICS
GRADE 12
AUGUST 2010

TIME: 3 HOURS

MARKS : 300

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages, and an Information booklet of 4 pages (i-iv). Please check that your question paper is complete. Please remove the insert from the middle of the question paper.

2. This question paper consists of TWO Modules:

   MODULE 1: CALCULUS AND ALGEBRA (200 marks) is compulsory

   MODULE 2: STATISTICS (100 marks) is your option module.

3. A non-programmable scientific calculator should be used where appropriate, unless
a particular question specifies otherwise.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. Round off your answers to 2 decimal digits, unless otherwise indicated.

7. Write all your answers in the separate Answer booklets provided.

MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

Use Mathematical induction to prove that $\sum_{i=1}^{n}(4i+7) = 2n^2 + 9n$ for all natural numbers n.

[12 marks]

QUESTION 2

2.1 Solve for x: $(\ln x)^2 = \ln e^2 + \ln x$ (6)

2.2 Sketch $y = \left|2 - e^x\right|$ (6)

[12 marks]

QUESTION 3

3.1 One root of a quadratic equation is $x = 3 - 5i$. Give the quadratic equation in the form $ax^2 + bx + c = 0$ (6)

3.2 Hence, or otherwise, solve $x^3 - x^2 + 4x + 170 = 0$ given that $x = 3 - 5i$ is a solution (4)

[10 marks]

QUESTION 4

Given that $\dfrac{1003}{1002 \times 1001^2} = \dfrac{A}{1002} + \dfrac{B}{1001} + \dfrac{C}{1001^2}$

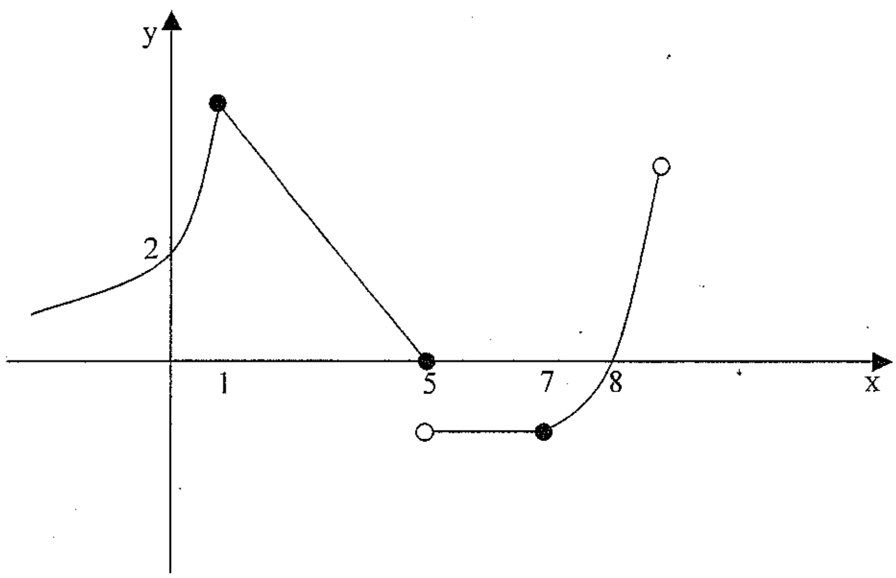
find the integer values of A, B and C respectively.

(*Hint: let x = 1001 and use partial fractions*)

[11 marks]

QUESTION 5

Given: $f(x)=\begin{cases} 3^x+1 & \text{if} \quad x\leq 1 \\ 5-x & \text{if} \quad 1<x\leq 5 \\ -1 & \text{if} \quad 5<x<7 \\ (x-7)^2-1 & \text{if} \quad 7\leq x<9 \end{cases}$



With reference to *f(x)* and its graph:

5.1 Give the domain and range of *f(x)*. (4)

5.2 Prove that *f(x)* is continuous at $x = 1$. (6)

5.3 Identify the type of discontinuity that exists at $x = 5$. (2)

5.4 Prove that f(x) is differentiable at $x = 7$. (6)

18 marks

QUESTION 6

Find:

6.1 $\lim_{x\to 0} \dfrac{\sin 3x}{x\cos 2x}$ (6)

6.2 $\lim_{x\to\infty} \dfrac{x^2+1}{2-3x-4x^2}$ (4)

6.3 $\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$ (6)

16 marks

QUESTION 7

Find the derivatives of the following functions: (DO NOT simplify your answers)

7.1 $f(x) = x^3(3x+1)$ (4)

7.2 $g(x) = \frac{x^3-1}{x^2+2}$ (6)

10 marks

QUESTION 8

The functions *f* and *g* are defined as follows:

$f(x) = \sin(\tan(2x))$
$g(x) = \sin x . \tan 2x$

Calculate, showing all necessary working:

(i) $f'(\pi)$ (5)

(ii) $g'(\frac{\pi}{3})$ (7)

12 marks

QUESTION 9

$f(x) = (2-3x)^{20}$

9.1 Write down the first, second and third derivatives of *f(x)*. (6)

9.2 Now write down a general formula for the n-th derivative of *f(x)* where n is a natural number. (6)

12 marks

QUESTION 10

10.1 If $x^m.y^n = k$, prove that $\frac{dy}{dx} = \frac{-my}{nx}$ (by using implicit differentiation). (6)

10.2 Hence find the equation of the tangent to $x^2y^3 = -8$ at the point where $x = 1$ (8)

14 marks

QUESTION 11

Assume that the functions $f(x), g(x)$ and $h(x)$ are continuous and differentiable for all real values, as are their derivatives. In the table below, the functions are evaluated at a point where $x = a$. However, two of the values are omitted.

It is also given that $h(x) = \frac{f(x)}{g(x)}$

Showing all calculations that lead to your answers, calculate the value of:

11.1 $g(a)$ (3)

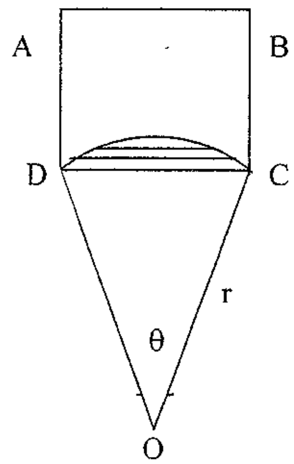
11.2 $h'(a)$ (7)

$f(a)$	1
$f'(a)$	2
$g(a)$	?
$g'(a)$	6
$h(a)$	$-\frac{1}{3}$
$h'(a)$	?

10 marks

QUESTION 12

The diagram shows a sector of a circle with centre O and radius r, and a chord DC which subtends an angle of θ radians at O, where $0 < \theta < \pi$.



A square ABCD is drawn.

(a) If the area of the shaded segment is exactly one eighth of the area of the square (i.e: $8 \times$ Area of Segment = area of square)

show that :

$$2\theta - 2\sin\theta + \cos\theta - 1 = 0 \qquad (8)$$

(b) Evaluate $2\theta - 2\sin\theta + \cos\theta - 1$ if

(1) $\theta = 1$ (2)

(2) $\theta = 2$ (2)

(c) Hence, using a suitable initial approximation, use the Newton Rhapson formula to find the value of θ correct to 4 decimal places. (8)

20 marks

QUESTION 13

The function f is given by $f(x) = \dfrac{x^2 + 4x + 3}{x + 2}$

13.1 Show that $f(x)$ can be written as $x + 2 - \dfrac{1}{x+2}$ (4)

13.2 Hence, or otherwise, find

13.2.1 $f'(x)$ (4)

13.2.2 the values of x for which the function is decreasing. (ie $f'(x) < 0$) (2)

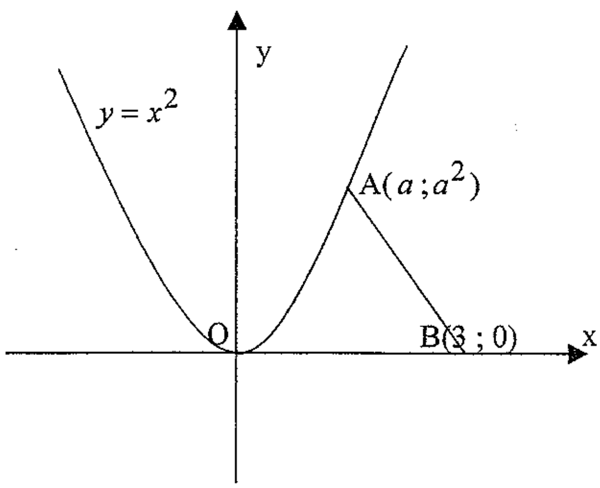
13.2.3 $f''(x)$ (2)

13.3 Sketch the curve $y = f(x)$, clearly showing asymptotes (vertical; horizontal; oblique), intercepts with the axes, stationary points and points of inflection (if any).
All relevant working must be shown.
(19)

31 marks

QUESTION 14

14.1 Show that if $A(a\,;a^2)$ is that point on the curve of $y = x^2$ which is the minimum distance/ closest to $B(3\,;0)$, then $2a^3 + a - 3 = 0$ (7)



14.2 Hence find the value of a which minimizes the distance AB (5)

12 Marks

MODULE 2 STATISTICS

QUESTION 1

Mrs. A. Nal, a Mathematics teacher gives her Grade 12 class from 2010, the exact same test that she gave her Grade 12 class in 2009. She then examines whether there is a significant difference between the means of the two normal distributions at the 5% significance level.

Class	Number in class	Mean	Variance
Grade 12 2009 (X)	30	75,5	100
Grade 12 2010 (Y)	28	78,2	110,25

If we state the null hypothesis in words we would say "there is no difference between the means of the two year groups". We would write $H_0 : \mu_X = \mu_Y$

1.1 Write down the alternate hypothesis (2)

1.2 Determine the acceptance region for Z. A sketch will be helpful (4)

1.3 Use a suitable formula, to determine whether we would reject the null hypothesis in favour of the alternative hypothesis. (10)

16 marks

QUESTION 2

Give your answers to the following correct to 4 decimal places.

2.1 Using your calculator (Paper 3 methods) find the equation of the least squares regression line of y on x. (4)

2.2 An alternative method for finding the value of B in $y = A + Bx$ is to use a formula

$$B = \frac{(n)(\sum xy) - (\sum x)(\sum y)}{(n)(\sum x^2) - (\sum x)^2}$$

Evaluate B using the following information:

$n = 7; \sum x = 38; \quad \sum y = 89; \quad \sum x^2 = 270; \quad \sum y^2 = 1147; \quad \sum xy = 495$ (6)

x	1	2	4	6	7	8	10
y	10	14	12	13	15	12	13

2.3 Write down the correlation coefficient, r, for this data. (4)

14 marks

QUESTION 3

A doctor knows from experience that 15% of the patients who are given a certain medicine will have side effects. If 18 people are given this medication what is the probability that exactly 4 people will have side effects?

8 marks

QUESTION 4

A particular electrical component is shipped in batches of twenty per box. Testing to determine whether an item is defective is very costly, and in order to minimize the number of defective items shipped to customers, a sampling scheme is devised. Five items are to be randomly selected from a batch, and if more than one item is defective, the batch is rejected. If a batch of twenty contains four defective components, find the probability of rejecting the batch if the sampling is done without replacement.

12 marks

QUESTION 5

The time taken to complete a mountain bike race is normally distributed with a mean of 4 hours 25 minutes and a standard deviation of 47 minutes.
A sample of 200 cyclists is taken.

How many cyclists will take between 4 hours and 5 hours 30 minutes to complete the race?

10 marks

QUESTION 6

For several years, a teacher has kept records on how long it takes learners to solve a rather difficult question.

If 64 learners (randomly selected) took on average 32,5 minutes with a variance of 10,89 minutes, construct a 99% confidence interval for the true average time it takes a learner to solve this problem?

8 marks

QUESTION 7

It is believed that the proportion of women in Gauteng who tested HIV + is 0,2245.

A 95% confidence interval is worked out to be (0,192 ; 0,257)

Calculate the number of women that were tested. **[10 marks]**

YOUR EARLY CHRISTMAS PRESENT:

$$p - Z\sqrt{\frac{p(1-p)}{n}} \leq \pi \leq p + Z\sqrt{\frac{p(1-p)}{n}}$$

QUESTION 8

If A and B are events in a sample space with $P(A) = 0{,}16$, $P(B) = 0{,}25$ and $P(A \cap B) = 0{,}1$

(a) Find

(1) $P(A \cup B)$ (6)

(2) $P(A \cap B')$ (4)

(b) Are A and B independent events? Give a reason for your answer. (4)

[14 marks]

QUESTION 9

The number of customers, X, in a shop at 10:00 has a probability distribution given by:

$P(X = x) = \frac{1}{k}(2x+1)$, where $x = 0; 1; 2; 3$ and k is a constant.

Find the value of k. **[8 marks]**