NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

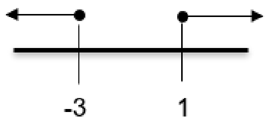
The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)(1)	$px^2 + 2x - 3 = 0$ $x = \dfrac{(-2) \pm \sqrt{(2)^2 - 4(p)(-3)}}{2(p)}$ $x = \dfrac{-2 \pm \sqrt{4 + 12p}}{2p}$ $x = \dfrac{-2 \pm 2\sqrt{1 + 3p}}{2p}$ $x = \dfrac{-1 \pm \sqrt{1 + 3p}}{p}$	✓ $b = 2$ ✓ $x = \dfrac{-2 \pm \sqrt{4 + 12p}}{2p}$ ✓ simplified solution (3)
(a)(2)	Non-real roots for: $1 + 3p < 0$ $p < -\dfrac{1}{3}$	✓ Correct method/ $\Delta < 0$ ✓ $p < -\dfrac{1}{3}$ (2)
(b)	$\sqrt{x - 2} + 4 = x$ $(x - 2) = (x - 4)^2$ $x - 2 = x^2 - 8x + 16$ $x^2 - 9x + 18 = 0$ $x = 6$ or $x = 3$ n/v for $x = 3$	✓ Isolate surd ✓ $x^2 - 4x + 4$ ✓ $x^2 - 9x + 18$ ✓ factors ✓ answer with selection (5)
(c)	$(x + 3)(x - 1) \geq 0$ Crit. values: –3 ; 1  $x \leq -3$ or $x \geq 1$	✓ Factors/critical values ✓ Number line/graph ✓ $x \leq -3$ or $x \geq 1$ (3)

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QUESTION 2

(a)	$x^{\frac{2}{3}} = 4$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ $x = \pm 8$ **Alternate:** $\sqrt[3]{x^2} = 4$ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x^2 = 64$ $x = 8$ or $x = -8$	$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ ✓ $x = 8$ ✓ $x = -8$ ✓ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x = 8$ ✓ (3)
(b)	$x^2 + 1 = x - y$ Sub: $y = 2 - 3x$ $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1$ or $x = 3$ When $x = 1$; $y = -1$ When $x = 3$; $y = -7$ **Alternate:** $y = 2 - 3x$ ... eq 1 $3^{x^2+1} = 3^{x-y}$ ... sub. eq 1 $3^{x^2+1} = 3^{x-(2-3x)}$ $3^{x^2+1} = 3^{4x-2}$ $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1$ or $x = 3$ When $x = 1$; $y = -1$ When $x = 3$; $y = -7$	$x^2 + 1 = x - y$ ✓ Sub: $y = 2 - 3x$ ✓ $x^2 + 1 = 4x - 2$ ✓ $x = 1$ ✓ $y = -1$ ✓ $x = 3$ ✓ $y = -7$ ✓ $3^{x^2+1} = \frac{3^x}{3^y}$ ... sub. eq 1 ✓ $3^{x^2+1} = 3^{4x-2}$ ✓ $x^2 + 1 = 4x - 2$ ✓ $x = 1$ ✓ $y = -1$ ✓ $x = 3$ ✓ $y = -7$ ✓ (7)
(c)	$A = P(1+i)^n$ $25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ $\frac{5}{4} = (1{,}04)^n$ $n = \log_{1{,}04}\left(\frac{5}{4}\right)$ $n \approx 5{,}7$ years After 6 years	$25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ ✓ $n = \log_{1{,}04}\left(\frac{5}{4}\right)$ ✓ $n \approx 5{,}7$ years ✓ 6 years ✓ (4)

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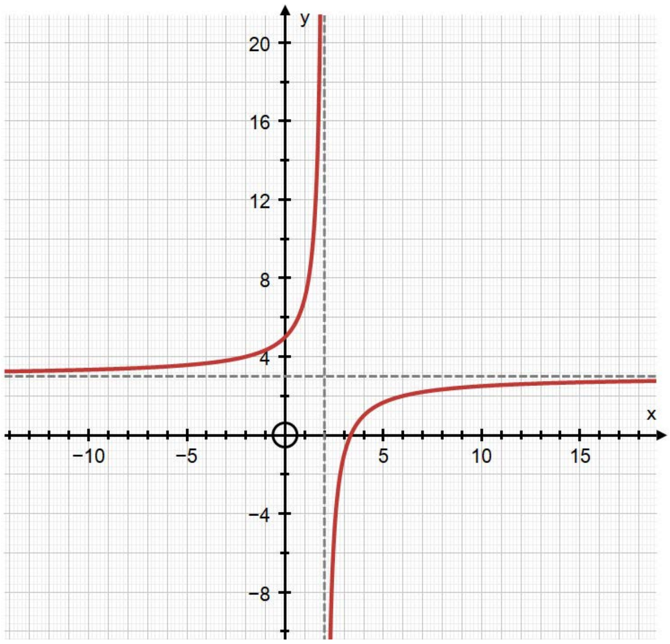
QUESTION 3

(a)	$f(0)=3-\frac{4}{0-2}$ $f(0)=5$	$f(0)=5$ ✓ (1)
(b)	$3-\frac{4}{x-2}=0$ $3(x-2)-4=0$ restr. $x\neq 2$ $3x-6-4=0$ $x=\frac{10}{3}$ $x=3\frac{1}{3}$	$3(x-2)-4=0$ ✓ $x=3\frac{1}{3}$ ✓ (2)
(c)	 	Shape ✓ Vertical Asymptote ✓ Horizontal Asymptote ✓ Intercepts ✓✓ (5)
(d)(1)	$f(x+p)=3-\frac{4}{x+p-2}$ $f(x+p)=-\frac{4}{[x+(p-2)]}+3$	$f(x+p)=3-\frac{4}{x+p-2}$ ✓ (1)
(d)(2)	Graph of *f* will shift *p* units to the right	Explanation ✓ (1)

(e)(1)	For $f^{-1}(x)$: $x = 3 - \frac{4}{y-2}$ $x = 3 - \frac{4}{y-2}$ $\frac{4}{y-2} = 3 - x$ $4 = (3-x)(y-2)$ $4 = 3y - 6 - xy + 2x$ $3y - xy = 4 + 6 - 2x$ $y(3-x) = 10 - 2x$ $y = \frac{10-2x}{3-x}$ $\therefore f^{-1}(x) = \frac{10-2x}{3-x}$ **Alternate final answer:** $f^{-1}(x) = \frac{2x-10}{x-3}$	$x = 3 - \frac{4}{y-2}$ ✓ $4 = (3-x)(y-2)$ ✓ $\therefore f^{-1}(x) = \frac{10-2x}{3-x}$ ✓ **Alternate final answer:** $f^{-1}(x) = \frac{2x-10}{x-3}$ (3)
(e)(2)	Domain of $f^{-1}(x)$: $x \in R$; $x \neq 3$	$x \in R$; $x \neq 3$ ✓ (1)

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QUESTION 4

(4)(a)	$ar^2 = 7$ $ar^5 = -2\,401$ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ $\therefore r^3 = -343$ $\therefore r = -7$ $T_n = a(-7)^{n-1}$ $T_3 = a(-7)^{3-1} = 7$ $a = \frac{7}{49}$ $\therefore a = \frac{1}{7}$	$ar^2 = 7$ ✓ $ar^5 = -2\,401$ ✓ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ ✓ $r = -7$ ✓ $a = \frac{1}{7}$ ✓ (5)
(4)(b)(1)	3 7 15 27 Sequence 4 8 12 First Difference 4 4 Constant second differ	Sequence ✓✓ First Difference ✓ Constant second differ ✓ (4)
(b)(2)	$2a = 4 \quad \therefore a = 2$ $3a + b = 4 \quad \therefore \quad b = -2$ $a + b + c = 3 \quad \therefore \quad c = 3$ $T_n = 2n^2 - 2n + 3$ **Alternate:** $T_n = 7(n-1) - 3(n-2) + \frac{(n-1)(n-2)}{2} \times (4)$ $T_n = 7n - 7 - 3n + 6 + (n^2 - 3n + 2)(2)$ $T_n = 2n^2 - 2n + 3$	Determining differences ✓ $a = 2$ ✓ $b = -2$ ✓ $c = 3$ ✓ Determining differences ✓ $a = 2$ ✓ $b = -2$ ✓ $c = 3$ ✓ (4)

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QUESTION 5

(a)	$g(x)=\log_t x$ sub. $(2;-1)$ $-1=\log_t 2$ $t^{-1}=2$ $t=\frac{1}{2}$	$-1=\log_t 2$ ✓ $t=\frac{1}{2}$ ✓ (2)
(b)	X-int. of normal/standard log graph is always: $(1;0)$ since $\log_t 1=0$ $\therefore C(1;0)$ **Alternate:** For Co-ord. of C: X-int, let $y=0$ $y=\log_{\frac{1}{2}} x$ $0=\log_{\frac{1}{2}} x$ $x=\left(\frac{1}{2}\right)^0$ $x=1$ $\therefore C(1;0)$	$\therefore C(1;0)$ ✓ $\therefore C(1;0)$ ✓ (1)
(c)	$f(x)=2p^x+q$ $q=-1$ since asymptote passes through $A(2;-1)$ $f(x)=2p^x-1$ ... sub. $(1;0)$ $0=2p^1-1$ $\therefore p=\frac{1}{2}$	$q=-1$ ✓ $0=2p^1-1$ ✓ $p=\frac{1}{2}$ ✓ (3)
(d)	D is the y-int of f: let $x=0$ $f(x)=2\times\left(\frac{1}{2}\right)^x-1$ ... sub. $x=0$ $y=2\times\left(\frac{1}{2}\right)^0-1$ $y=1$ $\therefore D(0;1)$	$y=2\times\left(\frac{1}{2}\right)^0-1$ ✓ $\therefore D(0;1)$ ✓ (2)
(e)	$f(x)=2\left(\frac{1}{2}\right)^x-1$ ... sub. $B(2;y)$ $f(x)=2\left(\frac{1}{2}\right)^2-1$ $f(x)=y=-\frac{1}{2}$ Length of AB $=\frac{1}{2}$	$f(x)=2\left(\frac{1}{2}\right)^2-1$ ✓ Length of AB $=\frac{1}{2}$ ✓ (2)
(f)	Range of f: $y>-1$	$y>-1$ ✓ (1)

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QUESTION 6

(a)	$f(x)=1-2x+x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{1-2(x+h)+(x+h)^2-(1-2x+x^2)}{h}$ $f'(x)=\lim_{h\to 0}\frac{1-2x-2h+x^2+2xh+h^2-1+2x-x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2h+2xh+h^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-2+2x+h)}{h}$ $f'(x)=\lim_{h\to 0}(-2+2x+h)$ $2x-2$	✓ $1-2(x+h)+(x+h)^2$ $-(1-2x+x^2)$ ✓ Squaring and distributing ✓ Factorisation ✓ Notation ✓ Sub. to get: $2x-2$ (5)
(b)	$y=x^{10}+10x$ $\frac{dy}{dx}=10x^9+10$	$10x^9$ ✓ 10 ✓ (2)
(c)	$y=\frac{5}{x^3}+\frac{x^{\frac{1}{2}}}{x^3}$ $y=5x^{-3}+x^{-\frac{5}{2}}$ $\frac{dy}{dx}=-15x^{-4}-\frac{5}{2}x^{-\frac{7}{2}}$	$y=5x^{-3}+x^{-\frac{5}{2}}$ ✓✓ $\frac{dy}{dx}=-15x^{-4}-\frac{5}{2}x^{-\frac{7}{2}}$ ✓✓ (4)

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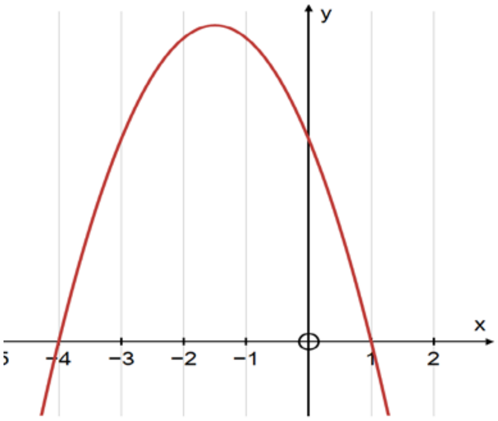
76 marks

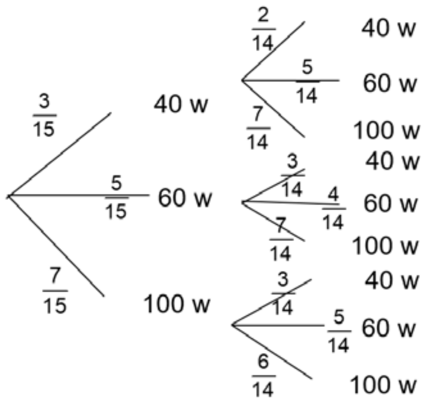
SECTION B

QUESTION 7

(a)	For: $x < -4$ and $x > 1$	$x < -4$ ✓ $x > 1$ ✓ (2)
(b)		Shape ✓ X-Intercepts ✓✓ (3)
(c)	$k > p$ or $k < q$	$k > p$ ✓ $k < q$ ✓ (2)
(d)	$x > -1\frac{1}{2}$	$x > -1\frac{1}{2}$ ✓✓ (2)

[9]

QUESTION 8

(a)(1)	8^6	8^6 ✓ (2)
(a)(2)	$=20\,160$	$8\times7\times6\times5\times4\times3$ ✓ 20 160 ✓ (2)
(b)(1)		$\frac{3}{15}$; $\frac{5}{15}$ and $\frac{7}{15}$ ✓ $\frac{\square}{14}$ ✓ Branches with correct values ✓ (3)
(b)(2)	$\left(\frac{5}{15}\times\frac{7}{14}\right)+\left(\frac{7}{15}\times\frac{5}{14}\right)$ $=\frac{1}{3}$	$\left(\frac{5}{15}\times\frac{7}{14}\right)$ ✓ $\left(\frac{7}{15}\times\frac{5}{14}\right)$ ✓ $=\frac{1}{3}$ ✓ (3)
(c)	$P(A\cap B)=P(A)\times P(B)$ $\therefore P(A\cap B)=0{,}08\times0{,}02$ $=0{,}0016$ but $P(AUB)=P(A)+P(B)-P(A\cap B)$ $\therefore P(AUB)=0{,}08+0{,}02-0{,}0016$ $=0{,}0984$ **Alternate:** P(at least one win) $=$P(one or more wins) $=1-$P(no wins) $=1-P(L)\times P(L)$ $=1-0{,}98\times0{,}92$ $=0{,}0984$	$\therefore P(A\cap B)=0{,}0016$ ✓ $P(AUB)=P(A)+P(B)-P(A\cap B)$ ✓ $\therefore P(AUB)=0{,}08+0{,}02-...$ ✓ $=0{,}0984$ ✓ $=1-$P(no wins) ✓ 0,98 ✓ 0,92 ✓ $=0{,}0984$ ✓ (4)

[14]

QUESTION 9

(a)	$a = 725$ $b = 190$	$a = 725$ ✓ $b = 190$ ✓ (2)
(b)	$h = k(x-a)^2 + b$ $h = k(x-725)^2 + 190$ sub. $(0;315)$ $315 = k(0-725)^2 + 190$ $k = \frac{1}{4\,205}$ $h = \frac{1}{4\,205}(x-725)^2 + 190$ ... sub. $(x;210)$ $210 = \frac{1}{4\,205}(x-725)^2 + 190$ $x = 1015$ or $x = 435$ Therefore the horizontal distance of hygrometer from the left tower is 435 m.	$h = k(x-725)^2 + 190$ ✓ $k = \frac{1}{4\,205}$ ✓ $210 = \frac{1}{4\,205}(x-725)^2$ $+190$ ✓ $x = 1015$ or $x = 435$ ✓✓ Therefore the horizontal distance of hygrometer from the left tower is 435 m. ✓ (6)

[8]

QUESTION 10

(a)	$F = 8\ 755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times4)}-1}{\dfrac{6,7}{400}}\right]$	$\left(1+\dfrac{6,7}{400}\right)$ ✓
	$F = 205\ 973,485$	Correct ***X*** in correct formula ✓
	Total cost of shares $= 8\ 755 \times 4 \times 5$ Total cost of shares $= 175\ 100$	$F = 205\ 973,485$ ✓ $175\ 100$ ✓
	Total Profit $= 30\ 873,485$	$30\ 873,485$ ✓
	% Profit $= \dfrac{30\ 873,485}{175\ 100} \times 100$ $= 17,6319\,\%$ $\approx 17,6\%$	$\approx 17,6\%$ ✓
	Alternate:	
	$F = 8\ 755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times4)}-1}{\dfrac{6,7}{400}}\right]$	$\left(1+\dfrac{6,7}{400}\right)$ ✓ Correct ***X*** in correct formula ✓
	$F = 205\ 973,485$	$F = 205\ 973,485$ ✓
	Total cost of shares $= 8\ 755 \times 4 \times 5$ Total cost of shares $= 175\ 100$	$175\ 100$ ✓
	$\therefore$ %Profit $= \dfrac{205\ 973,485}{175100}$ $= 1,176319$ $\therefore 17,6\%$	✓ % Profit $= \dfrac{205\ 973,485}{175\ 100}$ $\approx 17,6\%$ ✓ (6)

(b)	$300\,000 = x\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(15\times12)}}{\dfrac{9,5}{1200}}\right]$ $x = 3\,132,674049$ Balance of loan $= A - F$ $A = 300\,000\left(1+\dfrac{9,5}{1200}\right)^{12\times5}$ $A = 481\,502,8408$ $F = 3\,132,674049\left[\dfrac{\left(1+\dfrac{9,5}{1200}\right)^{(12\times5)}-1}{\dfrac{9,5}{1200}}\right]$ $F = 239\,405,9954$ Balance of loan $=(481\,502,8408)-(239\,405,9954)$ $= 242\,096,8454$ $\approx 242\,096,85$ **Alternate:** $P = 3\,132,674049\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(10\times12)}}{\dfrac{9,5}{1200}}\right]$ $P = 242\,096,8454$ $\approx 242\,096,85$ No, there would be a shortfall of R36 123,36	Use of correct formula ✓ $\left(1+\dfrac{9,5}{1200}\right)$ ✓ $n = 180$ in Pv-formula ✓ $x = 3\,132,674049$ ✓ $n = 120$ in Pv-formula ✓ $P = 242\,096,8454$ ✓ No, ✓ shortfall of R36 123,36 ✓ (8)

[14]

QUESTION 11

(a)	$\sum_{i=1}^{\infty}\frac{k}{2^i}+\sum_{i=1}^{10}2^{2i}>1000\,000$ Working with: $\sum_{i=1}^{\infty}\frac{k}{2^i}$ $T_1=\frac{k}{2}$; $T_2=\frac{k}{4}$; $T_3=\frac{k}{8}$ Common ratio: $\frac{k}{4}\div\frac{k}{2}$ $r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}$ for $-1<r<1$ $S_\infty=\frac{\frac{k}{2}}{1-\frac{1}{2}}$ $S_\infty=k$ Working with: $\sum_{i=1}^{10}2^{2i}$ $T_1=2^2$; $T_2=2^4$; $T_3=2^6$ Common ratio: $r=4$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq1$ $S_{10}=\frac{4\left(4^{10}-1\right)}{4-1}$ $S_{10}=1\,398\,100$ $\therefore\sum_{i=1}^{\infty}\frac{k}{2i}+\sum_{i=1}^{10}2^{2i}>1\,000\,000$ can be rewritten as $k+1\,398\,100>1000\,000$ $k>-398\,100$ $\therefore k=-398\,099\quad(k\in\mathbb{Z})$	$r=\frac{1}{2}$ ✓ Correct substitution into correct formula to get $S_\infty=k$ ✓ $r=4$ ✓ Correct substitution into correct formula to get $S_{10}=1\,398\,100$ ✓ $k+1\,398\,100>1\,000\,000$ ✓ $\therefore k=-398\,099\quad(k\in\mathbb{Z})$ ✓ (6)
(b)(1)	$5+\frac{15}{2}+10+...+\frac{505}{2}$ Common difference of $\frac{5}{2}$; series is arithmetic $T_n=a+(n-1)d$ $\frac{505}{2}=5+(n-1)\left(\frac{5}{2}\right)$ $250=\frac{5}{2}n$ $n=100$	$\mathrm{d}=\frac{5}{2}$ ✓ Correct substitution in the correct formula ✓ $n=100$ ✓ (3)

(b)(2)	Middle 30 terms would be: T_{36} to T_{65} $T_{36} = 5 + (35)\left(\frac{5}{2}\right)$ $T_{36} = \frac{185}{2}$ Let $a = \frac{185}{2}$; $d = \frac{5}{2}$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{30} = \frac{30}{2}\left[2\left(\frac{185}{2}\right) + (29)\left(\frac{5}{2}\right)\right]$ $S_{30} = 3\,862{,}5$ **Alternate:** Middle 30 terms would be: T_{36} to T_{65} $T_{36} = 5 + (35)\left(\frac{5}{2}\right)$ $T_{36} = \frac{185}{2}$ $T_{65} = 5 + (64)\left(\frac{5}{2}\right)$ $T_{65} = 165$ $S_n = \frac{n}{2}(a + l)$ $S_{30} = \frac{30}{2}\left(\frac{185}{2} + 165\right)$ $S_{30} = 3\,862{,}5$	Middle 30: T_{36} to T_{65} ✓ $T_{36} = \frac{185}{2}$ ✓ Correct substitution into correct formula ✓ $S_{30} = 3\,862{,}5$ ✓ Middle 30: T_{36} to T_{65} ✓ $T_{36} = \frac{185}{2}$ ✓ Correct substitution into correct formula ✓ $S_{30} = 3\,862{,}5$ ✓ (4)

[13]

QUESTION 12

12	Let: $g(1)=h(1)$ $(1)^3-a(1)^2+6=2(1)^2+b(1)+3$ $1-a+6=2+b+3$ $a=2-b$ ... eq1 $g'(x)=3x^2-2ax$ $h'(x)=4x+b$ $g'(1)=h'(1)$ $3(1)^2-2a(1)=4(1)+b$ $3-2a=4+b$ ... sub eq1: $a=2-b$ $3-2(2-b)=4+b$ $b=5$ $a=-3$ $h(x)=2x^2+5x+3$ $h(1)=10$ Point of contact is: $(1;10)$	$g(1)=h(1)$ ✓ $a=2-b$ ... eq1 ✓ ✓ $g'(x)=3x^2-2ax$ $h'(x)=4x+b$ ✓ $g'(1)=h'(1)$ ✓ $2a+b=-1$ ✓ $b=5$ ✓ $(1;10)$ ✓ (8)

[8]

QUESTION 13

13	$8x+4x+4h=P$ $P=12x+4h$ $P-12x=4h$ $\therefore h=\frac{1}{4}P-3x$ $V=l\times b\times h$ $V=(2x)(x)(h)$ ... sub.: $h=\frac{1}{4}P-3x$ $V=(2x)(x)\left(\frac{1}{4}P-3x\right)$ $V=\frac{1}{2}x^2P-6x^3$ $V'=Px-18x^2$ $0=x\left(P-18x\right)$ $x=0$ or $x=\frac{P}{18}$ Hence, length of the box is $2x=\frac{P}{9}$ $\therefore$ length of box is $\frac{1}{9}P$ cm when the volume is a maximum.	$8x+4x+4h=P$ ✓ $h=\frac{1}{4}P-3x$ ✓ $V=(2x)(x)\left(\frac{1}{4}P-3x\right)$ ✓ $V=\frac{1}{2}x^2P-6x^3$ ✓ $V'=Px-18x^2$ ✓ $0=x\left(P-18x\right)$ ✓ $P=18x$ ✓ Length of the box is $2x$ and $P=18x$ ✓

[8]

74 marks

Total: 150 marks