NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2024

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

(a) $(3x-4)(x+3)(x+3)=0$ ✓

$x=\frac{4}{3}$ or $x=-3$ ✓✓

(b) No real Solutions ✓

(c) $\log_x 5 = 3$

$x^3 = 5$ ✓

$x=\sqrt[3]{5} = 1{,}7$ ✓

(d) $4^{x+1} + 2^{2x} = 40$

$2^{2x+2} + 2^{2x} = 40$ ✓

$2^{2x}(2^2+1) = 40$ ✓

$2^{2x} = 2^3$ ✓

$2x = 3$

$x=\frac{3}{2}$ ✓

(e) $\frac{8-4x}{x-2}=x$

$\frac{-4(x-2)}{(x-2)}=x$ ✓✓

$x=-4$ ✓

Alternative: $8-4x = x^2-2x \therefore x^2+2x-8=0$ ✓

Thus $(x+4)(x-2)=0$

$x=-4$ or $x \neq 2$ ✓✓

(f) (1) $(x+k)^2 = 2k+1$

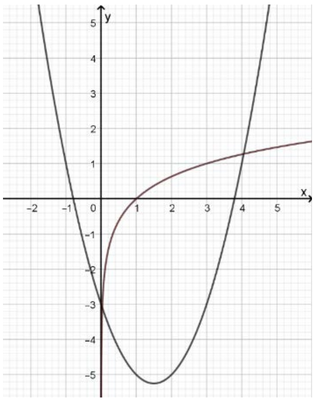
$x=-k\pm\sqrt{2k+1}$ ✓✓

ALT: $x^2 + 2kx + (k^2 - 2k - 1) = 0$ $x=\frac{-2k\pm\sqrt{4k^2-4\left(k^2-2k-1\right)}}{2}=\frac{-2k\pm\sqrt{4\left(2k+1\right)}}{2}$

(2) $k=\frac{3}{2}$ or $k=4$ or $k=\frac{15}{2}$ or $k=12$ ✓✓

QUESTION 2

(a)



✓ x-intercept (1; 0)

✓ Shape

✓ Asymptote x = 0

(b) $x = \log_3(y)$ ✓

$y = 3^x$ ✓

(c) (1) $y = \log_3(x - 1)$ ✓

(2) x = 1✓

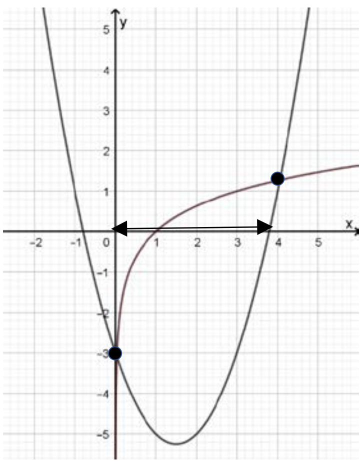
(d) (1) $f(x) = x^2 - 3x + \frac{9}{4} - 3 - \frac{9}{4}$ ✓

$f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{21}{4}$ ✓✓

(2) $x \in (-\infty; \infty)$ ✓ and $y \in [-\frac{21}{4}; \infty)$ ✓✓notation

(e) $x = 3$ ✓

(f)



✓✓

QUESTION 3

(a) $f(x) = x^2 - x$

$$f'(x) = \lim_{h \to 0}\left(\frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}\right) ✓$$

$$= \lim_{h \to 0}\left(\frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}\right) ✓$$

$$= \lim_{h \to 0}\left(\frac{h(2x + h - 1)}{h}\right) ✓$$

$$= \lim_{h \to 0}(2x + h - 1) ✓$$

$f'(x) = 2x - 1$ ✓✓notation

(b) $g(x) = 4x^3 - 2x^{-2} + \sqrt[8]{x}$

$g(x) = 4x^3 - 2x^{-2} + x^{\frac{1}{8}}$ ✓

$g'(x) = 12x^2 + 4x^{-3} + \frac{1}{8}x^{-\frac{7}{8}}$ ✓✓✓

(c) $h(x) = 3x^2 - 4x$

$h(1) = -1$ ✓

$h'(x) = 6x - 4$ ✓

$h'(1) = 2$ ✓

$y = 2x + c$ *Alternative*: $y + 1 = 2(x - 1) \therefore y = 2x - 3$

$-1 = 2(1) + c$ ✓

$c = -3$ ✓

Tangent @ $x = 1$

$y = 2x - 3$

QUESTION 4

(a) (1) $T_n = 15 + (n-1)(-4)$ ✓✓

$T_n = -4n + 19$ ✓

(2) $15 + 11 + 7 + 3 - 1 - 5 + \ldots - 2009 - 213$

$-4n + 19 = -213$ ✓

$-4n = -232$

$n = 58$ ✓

$S_n = \frac{58}{2}(2(15) + (58-1)(-4))$ ✓

$S_n = -5\ 742$ ✓

(b) $\dfrac{\frac{3}{5}}{1-\frac{3}{5}}$ ✓✓ $+ \dfrac{\frac{9}{25}\left(\left(\frac{3}{5}\right)^{19}-1\right)}{\frac{3}{5}-1}$ ✓✓✓

$= \frac{3}{2} + 0{,}8999945 \ldots$

$= 2{,}4$ ✓

QUESTION 5

(a) (1) 6! OR 720 ways ✓

(2) 4! X 3! ✓✓ or 4!3!

= 144 ways ✓

(b) DRESSES

$= \frac{7!}{3!2!}$ ✓✓✓

= 420 ways

QUESTION 6

(a) $1\ 500\ 000 = \frac{x\left(1-\left(1+\frac{0,11}{12}\right)^{-240}\right)}{\frac{0,11}{12}}$ ✓✓✓

x = R15 482,83 ✓

(b) $\text{B.O.} = 1\ 500\ 000\left(1+\frac{0,11}{12}\right)^{108} - \frac{15\ 482,83\left(\left(1+\frac{0,11}{12}\right)^{108}-1\right)}{\frac{0,11}{12}}$ ✓✓✓

B.O. = R1 182 585,03 ✓

Alternative: $15\ 482,83 \times \frac{\left(1-\left(1+\frac{0,11}{12}\right)^{-132}\right)}{\frac{0,11}{12}}$ ✓✓✓= 1 182 586,10✓

QUESTION 7

$$25000\left(1+\frac{0{,}15}{4}\right)^{32} \checkmark\checkmark + \checkmark \ \frac{600\left(\left(1+\frac{0{,}15}{4}\right)^{32}-1\right)}{\frac{0{,}15}{4}} \ \checkmark\checkmark\checkmark$$

R81 200,63 + R35 968,40

R117 169,03 ✓

QUESTION 8

(a) $f'(x) = -4x + 15$ ✓

$-4x + 15 = 3$ ✓

$x = 3$ ✓

$f(3) = -2(3)^2 + 15(3)$	Alternative: If $-2x^2 + 15x = 3x + c$
$f(3) = 27$	$\therefore\ 2x^2 - 12x + c = 0$
$27 = 3(3) + c$ ✓	$\Delta = 0$: $144 - 8c = 0$
$c = 18$	$\therefore\ c = 18$

$g(x) = 3x + 18$

$0 = 3x + 18$

$x = -6$ ✓

(b) (1) $h(x) = ax^3 + bx^2$.

$h'(x) = 3ax^2 + 2bx$ ✓

$h''(x) = 6ax + 2b$ ✓

$6a(-1) + 2b = 0$ ✓

$2b = 6a$ ✓

$40 = 8a + 4b$ Subbing in the point (2;40) ✓

$20 = 4a + 2b$

$20 = 4a + 6a$

$20 = 10a$

$a = 2$ ✓

$b = 6$ ✓

(2) $y = 2(-1)^3 + 6(-1)^2$.

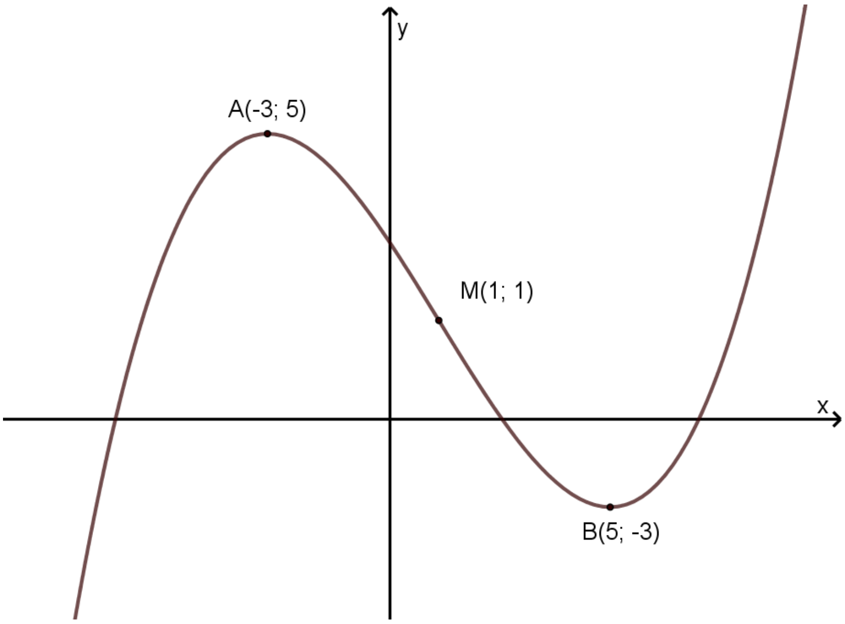
$m(-1; 4)$

$P(0;4)$ ✓

$PB = \sqrt{(40-4)^2 + (2-0)^2}$ ✓

$PB = \sqrt{1300} = 36{,}1$ km ✓

(c) (1)



✓ Shape ✓ A and B ✓ Three x-intercepts ✓POI

(2) $t > 3$ ✓ or $t < -5$ ✓

QUESTION 9

(a) $-\log_3 x + (6-1)\log_3 x = 12$ ✓

$4\log_3 x = 12$ ✓

$\log_3 x = 3$ ✓

$x = 3^3 = 27$ ✓

Alternative:

$-\log_3 x + (6-1)\log_3 x = 12$ ✓

$-\log_3 x + \log_3 x^5 = 12$ ✓

$\log_3 x^4 = 12$ ✓

$x^4 = 3^{12}$

$x = 27$ ✓

(b) $\dfrac{4x-3}{2x+6} = \dfrac{2x+6}{5x+1}$ ✓✓

$20x^2 - 11x - 3 = 4x^2 + 24x + 36$ ✓

$0 = 16x^2 - 35x - 39$

$x = 3$ or $x \neq -\dfrac{13}{16}$ ✓✓✓

$x = 3$ gives $r = \dfrac{3}{4}$ hence sequence will converge

QUESTION 10

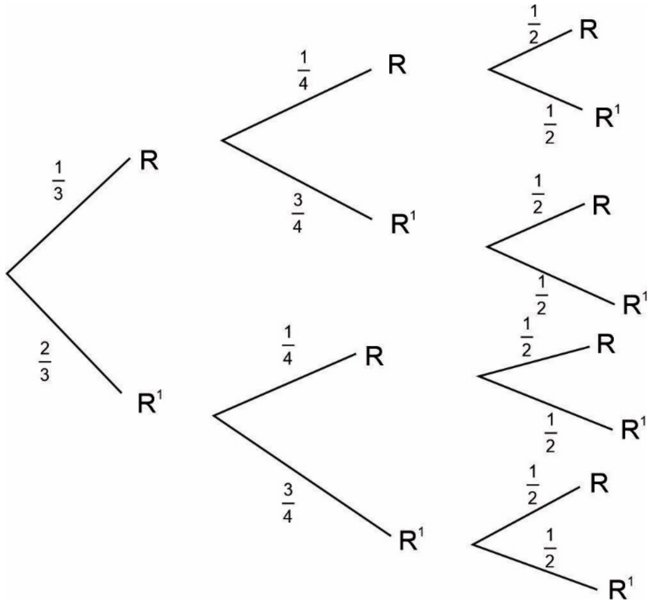
(a) (1) $3 \times 4 \times 2 \times 5 = 120$ ways ✓✓

(2) $1-\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$ ✓✓ subtract

$\frac{18}{24}$ or $\frac{3}{4}$

$= 0,75$ ✓

Alternative:



$$\frac{1}{3}\times\frac{1}{4}\times\frac{1}{2}\times 2+\frac{1}{3}\times\frac{3}{4}\times\frac{1}{2}\times 2+\frac{2}{3}\times\frac{1}{4}\times\frac{1}{2}\times 2+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{2}=\frac{3}{4}$$

ALT: From tree diagram – $\frac{1}{3}\times 1\times 1+\frac{2}{3}\times\frac{1}{4}\times 1+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{2}=\frac{1}{3}+\frac{1}{6}+\frac{1}{4}=\frac{3}{4}$

(b) P(Dart landing in the shaded region) = $\frac{1}{3}$ ✓

P(Dart landing in the smaller square) = $\frac{2}{3}$ ✓

The probability that one dart lands in the shaded area and the other does not

$\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}$ ✓✓

$=\frac{4}{9}$ ✓

QUESTION 11

(a) $\dfrac{10}{x^2-5x-6} \geq 0$

$\dfrac{10}{(x-6)(x+1)} \geq 0$ ✓

$x < -1$ or $x > 6$ ✓✓ ✓notation

(b) $\sqrt{2x+1}-2=y$ and $\dfrac{\sqrt{x}}{\sqrt{y}}=2$

$\dfrac{x}{y}=4$

$x=4y$ ✓

$\sqrt{2(4y)+1}-2=y$ ✓

$8y+1=y^2+4y+4$

$0=y^2-4y+3$ ✓

$0=(y-3)(y-1)$

$y=3$ or $y=1$ ✓

$2x+1=(1+2)^2$

x = 4 ✓

$2x+1=(3+2)^2$

x = 12 ✓

(4; 1) or (12; 3)

QUESTION 12

(a) (1) $f(x) = \dfrac{x-2+3}{x-2}$

$f(x) = \dfrac{3}{x-2} + 1$ ✓✓

$y = x + c$✓

$1 = 2 + c$ ✓ Sub in (2; 1)

$c = -1$✓

$y = x - 1$ ✓

ALT: $m = 1$ hence $y - 1 = x - 2$

(2) $\dfrac{3}{x-2} + 1 = x - 1$ ✓

$3 = (x-2)^2$ ✓

$x = 2 \pm \sqrt{3}$

$A\left(2-\sqrt{3};1-\sqrt{3}\right)$ ✓✓

ALT: $3 + x - 2 = (x-1)(x-2) \therefore x^2 - 4x + 1 = 0 \therefore x = \dfrac{4 \pm \sqrt{12}}{2}$

(b) $FE = y - 2x$ ✓

$2x + y - 2x + x + 2x + x + y = 20$

$4x + 2y = 20$ ✓

$y = 10 - 2x$

Area = $2x(y - 2x) + 2x(x)$

Area = $2xy - 2x^2$ ✓

Alt: Area = area bigger rectangle ABHF – area rectangle EDCH = $2xy - 2x^2$

Area = $2x(10 - 2x) - 2x^2$

Area = $20x - 6x^2$ ✓

$\frac{dA}{dx} = 20 - 12x$ ✓

$20 - 12x = 0$ ✓

$x = \frac{5}{3}$ ✓

Total: 150 marks