MATHEMATICS

(PAPER 2)

JUNE 2008

MEMORANDUM

TIME: 3 HOURS
MARKS: 150

education

Western Cape Education Department

NATIONAL STRATEGY FOR LEARNER ATTAINMENT

NATIONAL SENIOR CERTIFICATE

MEMO

This memo consists of **9** pages

Wiskunde/Mathematics **Graad/Grade 12** **Junie/June 2008**

Tyd/Time: 3 uur/hours **Vraestel/Paper 2** **Punte/Marks: 150**

VRAAG 1/QUESTION 1

1.1 $AC = \sqrt{8^2 + 4^2}$ ✓

$= 4\sqrt{5}$ ✓

1.2 $\frac{y-1}{x+3} = \frac{-3-1}{5+3} = -\frac{1}{2}$ ✓✓

Equation is $2y + x + 1 = 0$ ✓

1.3 At point of intersection of AC and BD: $2(2x-8) + x + 1 = 0$ ✓✓

$\therefore 4x - 16 + x + 1 = 0$

$\therefore 5x = 15$

$\therefore x = 3$

and $y = -2$ ✓✓

But the mid-point of BD is $\left(\frac{4+2}{2}; \frac{0-4}{2}\right) = (3;-2)$ ✓✓

And gradient of AC × gradient of BD = $-\frac{1}{2} \times 2 = -1$ ✓✓

Hence AC is the perpendicular bisector of BD

1.4 Area of kite ABCD = 2× area ΔACD ✓

$= 2 \times 0{,}5 \times 4\sqrt{5} \times \sqrt{(4-3)^2 + (0+2)^2}$ ✓✓

= 20 square units ✓

1.5 The inclination of AB = $\tan^{-1}\left(\frac{-4-1}{2+3}\right)$ ✓

$= 135^0$ ✓

1.6 The inclination of AD = $\tan^{-1}\left(\frac{1}{-7}\right) = 171{,}9$ (correct to 1 decimal place) ✓✓

$B\hat{A}D = 171{,}9^0 - 135^0 = 37^0$ (correct to nearest degree) ✓✓

[23]

VRAAG 2/QUESTION 2

2.1 Vol. of hemisphere = $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\left(\frac{4}{3}\pi(40)^3\right) = 134\ 041{,}29\ cm^3$ ✓✓

Vol. of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(40)^2(100) = 167\ 551{,}61\ cm^3$ ✓✓

$\therefore$ Volume of model = $301\ 592{,}9\ cm^3$ ✓

2.2 Surface area of cone = $\frac{1}{2}(4\pi r^2) = 2\pi(0{,}4)^2 = 1{,}01\ m^2$ ✓✓

Surface area of hemisphere = $\pi r^2 + \pi rH$

= $\pi(0{,}4)^2 + \pi(0{,}4)(\sqrt{(0{,}4)^2 + (1)^2})$ ✓

= $1{,}86\ m^2$ ✓

Total exterior area = $1{,}01 + 1{,}86 = 2{,}87\ m^2$ ✓

2.3 Mass of steel model = $2{,}87 \times 2{,}5 = 7{,}18\ kg$ ✓

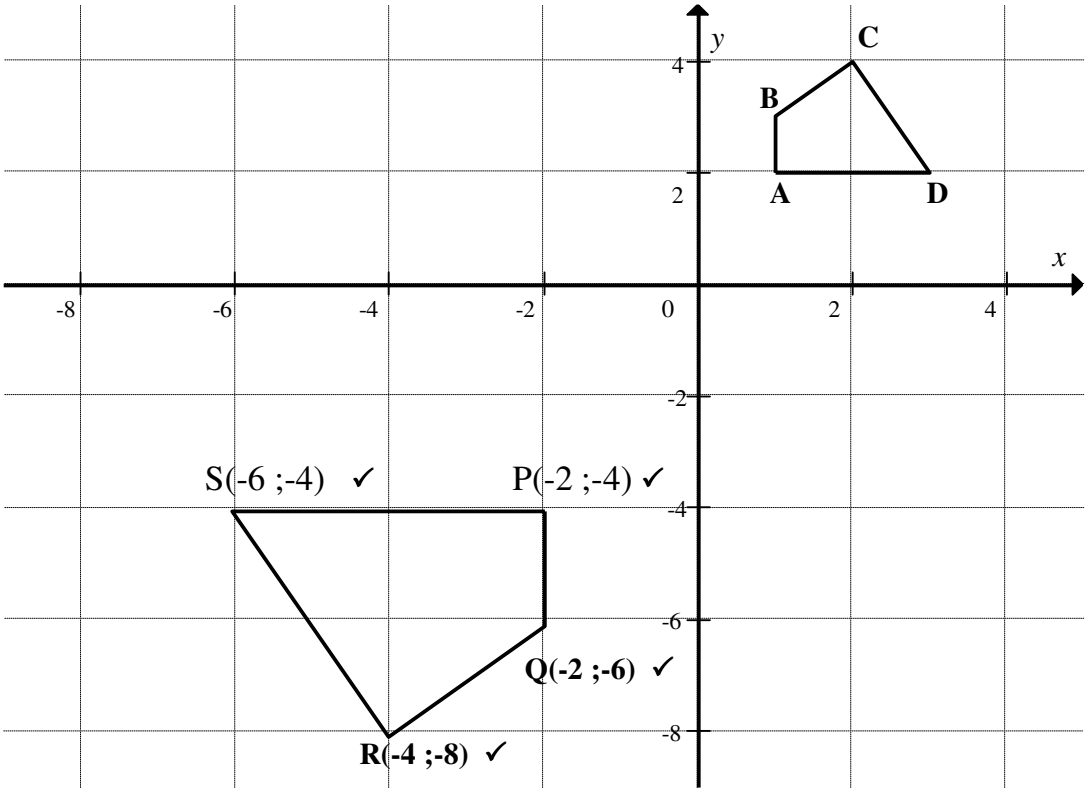
[11]

VRAAG 3/QUESTION 3

3.1.1 P($2\ ; -\sqrt{3}$) ✓✓

3.1.2 P($-\sqrt{3}\ ; 2$) ✓✓

3.2.1



3.2.2 $(x\ ;\ y) \rightarrow (-x\ ;\ -y)$ ✓✓

$(-x\ ;\ -y) \rightarrow (-2x\ ;\ -2y)$ ✓

$(x\ ;\ y) \rightarrow (-2x\ ;\ -2y)$ ✓

3.2.3 Area ABCD : area PQRS = 1 : 4 ✓

3.3 Let r = OP = OP$^{/}$

The x co-ordinate of $P^{/} = r\cos(\alpha + 30)°$ ✓

$x^{/} = r\cos(\alpha + 30)$

$= r(\cos\alpha.\cos30° - \sin\alpha.\sin30°)$ ✓

$= r\cos\alpha.\cos30° - r\sin\alpha.\sin30°$

$= r.\frac{x}{r}.\frac{\sqrt{3}}{2} - r.\frac{y}{r}.\frac{1}{2}$ ✓✓

$= \frac{\sqrt{3}}{2}x - \frac{y}{2}$ ✓

The y co-ordinate of $P^{/} = r\sin(\alpha + 30)°$

Similarly,

$y^{/} = r\sin(\alpha + 30)$ ✓

$y^{/} = y\cos30 + x\sin30°$ ✓

$= \frac{\sqrt{3}}{2}y + \frac{x}{2}$ ✓

3.4 $K^{/} = \left(\frac{\sqrt{3}}{2}x - \frac{y}{2}\ ;\ \frac{\sqrt{3}}{2}y + \frac{x}{2}\right)$

$K^{/} = \left(4.\frac{\sqrt{3}}{2} - 3.\frac{1}{2}\ ;\ 3.\frac{\sqrt{3}}{2} + 4.\frac{1}{2}\right)$

$K^{/} = (1{,}96\ ;\ 4{,}60)$ ✓✓

$L^{/} = \left(3.\frac{\sqrt{3}}{2} - 6.\frac{1}{2}\ ;\ 6.\frac{\sqrt{3}}{2} + 3.\frac{1}{2}\right)$

$L^{/} = (-0{,}40\ ;\ 6{,}70)$ ✓✓

[25]

VRAAG 4/QUESTION 4

4.1 $\dfrac{\sin(-\alpha)\cos(90^0-\alpha)}{\cos\alpha\cos(180^0+\alpha)} = \dfrac{(-\sin\alpha)(\sin\alpha)}{\cos\alpha(-\cos\alpha)}$ ✓✓✓

$= \dfrac{-\sin^2\alpha}{-\cos^2\alpha}$ ✓

$= \tan^2\alpha$ ✓

4.2.1 $\cos 27^0 = \sqrt{1-\sin^2 27^0} = \sqrt{1-t^2}$ ✓✓

4.2.2 $\tan 153^0 = -\tan 27^0$ ✓

$= -\dfrac{t}{\sqrt{1-t^2}}$ ✓✓

4.2.3 $\cos 243^0 = -\cos 63^0$ ✓

$= -\sin 27^0 = -t$ ✓

4.2.4 $\cos 54^0 = \cos(2\times 27^0)$ ✓

$= 1-2\sin^2 27^0 = 1-2t^2$ ✓✓

4.3 $\tan(3x+75^0) = -1$

$\therefore 3x+75^0 = -45^0 + k.180^0; \quad k \in Z$ ✓✓

$\therefore 3x = -120^0 + k.180^0$ ✓

$\therefore x = -40^0 + k.60^0; \quad k \in Z$ ✓✓

4.4 $\dfrac{\sin 15^0}{2} + \dfrac{\sqrt{3}\cos 195^0}{2} = \dfrac{\sin 15^0}{2} + \dfrac{\sqrt{3}\cos(180^0+15^0)}{2}$ ✓

$= \dfrac{1}{2}.\sin 15^0 + \dfrac{\sqrt{3}}{2}(-\cos 15^0)$

$= \sin 30^0 \sin 15^0 - \cos 30^0 \cos 15^0$ ✓✓

$= -\cos(30^0+15^0)$ ✓

$= -\dfrac{\sqrt{2}}{2}$ ✓

[25]

VRAAG 5/QUESTION 5

5.1 In ΔADC: $\cos\theta = \dfrac{r^2 + r^2 - k^2}{2r.r}$ ✓

$= \dfrac{2r^2 - k^2}{2r^2}$ ✓

5.2 In ΔABD: $\cos(180^0 - \theta) = \dfrac{r^2 + (2r)^2 - (2k)^2}{2.r.2r}$ ✓

$\therefore -\cos\theta = \dfrac{5r^2 - 4k^2}{4r^2}$ ✓

$\therefore \cos\theta = \dfrac{4k^2 - 5r^2}{4r^2}$ ✓

5.3 Hence $\dfrac{2r^2 - k^2}{2r^2} = \dfrac{4k^2 - 5r^2}{4r^2}$ ✓

$\therefore 4r^2 - 2k^2 = 4k^2 - 5r^2$

$\therefore 9r^2 = 6k^2$

and $k^2 = \dfrac{3}{2}r^2$ ✓

So $\cos\theta = \dfrac{2r^2 - \frac{3}{2}r^2}{2r^2} = \dfrac{1}{4}$ ✓

[8]

VRAAG 6/QUESTION 6

6.1 Bearing = 60^0 ✓

6.2 $AB^2 = 300^2 + 500^2 - 2.300.500.\cos 120^0$ ✓✓✓

$= 90\,000 + 250\,000 - 300\,000\left(-\dfrac{1}{2}\right)$ ✓✓

$= 490\,000$

$\therefore AB = 700$ metres

Race distance = $300 + 500 + 700 = 2\,200$ metres ✓

[7]

VRAAG 7/QUESTION 7

7.1 $\frac{20,7}{BD} = \cos 43,6°$ ✓

$BD = \frac{20,7}{\cos 43,6°}$ ✓

$BD = 28,6$ m ✓

7.2 $BE^2 = 28,6^2 + 28,1^2 - 2(28,6)(28,1)\cos 35,7°$ ✓✓

$= 302,2919056$ ✓

$BE = 17,4$ m ✓

7.3 Area $\Delta BEC = \frac{1}{2}(20,7)(17,4)\sin 63°$ ✓

$= 160,5\ m^2$ ✓

[9]

VRAAG 8/QUESTION 8

8.1 $\cos\frac{x}{2} = \sin(x - 30°)$

$\cos\frac{x}{2} = \cos(90° - x + 30°)$ ✓

$\frac{x}{2} = 120° - x + k.360°$ or $\frac{x}{2} = -120° + x + k.360°$; $k \in Z$ ✓✓

$\frac{3x}{2} = 120° + k.360°$ or $\frac{-x}{2} = -120° + k.360°$

$x = 80° + k.240°$ or $x = 240° - k.720°$ ✓✓

$x = 80°$; $-160°$ ✓✓

A(80° ; 0,766) and B(-160° ; 0,174)

8.2 $-160° < x < 80°$ ✓✓

[9]

VRAAG 9/QUESTION 9

$\bar{x} = 185\ g$ ✓

$\sigma = \sqrt{\sum_{i=1}^{8} \frac{(x_i - 185)^2}{8}} = 25,98\ g$ (correct to 2 decimal places) ✓

or 26 g (correct to the nearest g ✓✓

[4]

VRAAG 10/QUESTION 10

10.1 True: 50% of the data items are within the inter-quartile range compared to 68% within one standard deviation. ✓✓

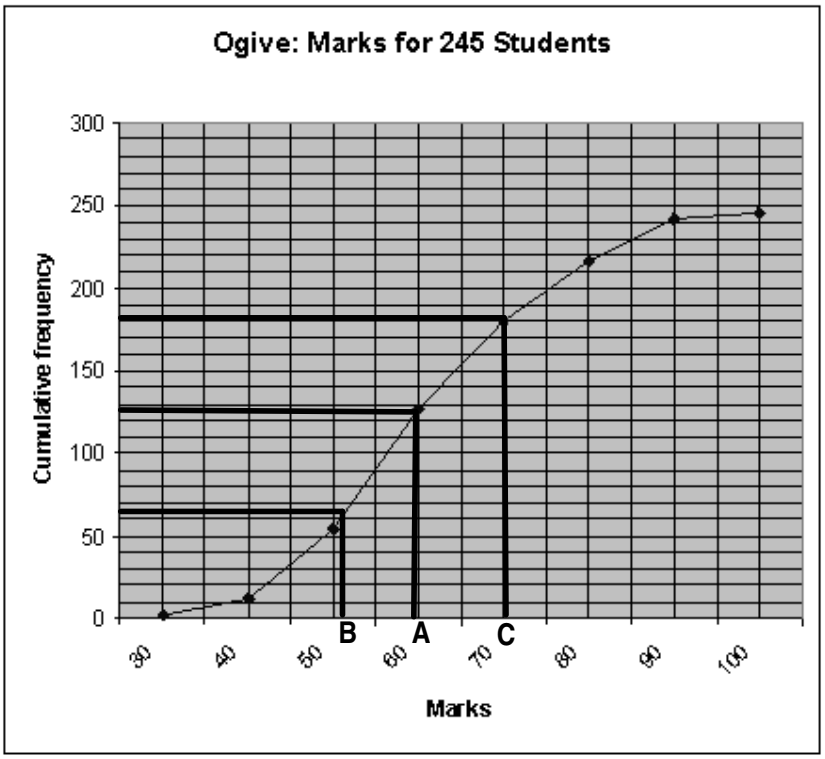
10.2 True: the data is spread more to the left of the median than to the right. ✓✓

10.3 False: the greater the time, the lower the water level, so the correlation is negative. ✓✓

[6]

VRAAG 11/QUESTION 11

11.1



✓✓✓✓
✓✓✓✓

11.2 Median ≈ 59% (read at A) ✓✓
Lower quartile ≈ 51 (read at B) ✓✓
Upper quartile ≈ 71% (read at C) ✓✓

11.3 $\text{Mean} = \dfrac{2\times15+10\times35+43\times45+72\times55+53\times65+37\times75+25\times85+3\times95}{245}$ ✓✓✓✓

$= 60{,}84\%$ (correct to 2 decimal places0 ✓✓

11.4 Mean $\approx 61\%$, median $\approx 59\%$ and mode 55%. Because all these values are approximate, and the differences are not significant, we can say that the distribution is fairly symmetric (the data is not skewed) ✓✓✓

[23]

TOTAL: [150]